On the Moderation of Mechanisms:  
A Conceptual Overview of Conditional Process Analysis

Andrew F. Hayes  
Professor of Quantitative Psychology  
The Ohio State University  
Department of Psychology

These are slides at:  
www.afhayes.com
I will ✔ and will not □

- do a brief review of the analysis of indirect and conditional effects.
- do a conceptual introduction to “conditional process analysis.”
- define the conditional indirect and direct effect.
- do one simple example, with relevant computer output.
- talk about some interesting extensions of basic principles.
- speak mostly in abstractions. You can fill in the blanks concretely.
- turn you into a conditional process analysis expert.
- teach you how to estimate such models in your chosen software.
- get all your questions answered.
- leave you with many new questions unanswered.
- point you toward where you can learn more.

My objective is primarily to whet your appetite for learning more. Knowing what is possible analytically can influence how we think about problems theoretically.
“Conditional process analysis” is a modeling strategy undertaken with the goal of describing the conditional or contingent nature of the mechanism(s) by which a variable transmits its effect on another, and testing hypotheses about such contingent effects.

A melding of two ideas conceptually and analytically:

“Process analysis”, used to quantify and examine the direct and indirect pathways through which an antecedent variable $X$ transmits its effect on a consequent variable $Y$ through an intermediary $M$. Better known as “mediation analysis” these days.

“Moderation analysis” used to examine how the effect of an antecedent $X$ on an consequent $Y$ depends on a third moderator variable $M$ (a.k.a. “interaction”)

Mechanisms are quantified with indirect effects. Indirect effects can be moderated, meaning mechanisms can be contingent. We can model such contingencies using rudimentary linear modeling principles. It is not difficult once you learn the fundamentals.
A mediation model links a putative cause (X) to a presumed effect (Y) at least in part through an intermediary or “mediator” variable (M).

The “simple mediation” model

$X \rightarrow M \rightarrow Y$ is a causal chain of events. A mediator variable can be a psychological state, a cognitive process, an affective response, a biological change, or any other conceivable “mechanism” variable through which $X$ exerts an effect on $Y$. But it must be causally between $X$ and $Y$. 
“Indirect effect”

Using OLS or ML, with $Y$ as continuous:

$$\hat{Y} = i_1 + cX$$

$c = \text{“total effect” of } X \text{ on } Y$

Using OLS or ML, with $M$ as continuous:

$$\hat{M} = i_2 + aX$$

$$\hat{Y} = i_3 + c'X + bM$$

$c' = \text{“direct effect” of } X \text{ on } Y$

$ab = \text{“indirect effect” of } X \text{ on } Y \text{ through } M$

$c = c' + ab$

The indirect effect quantifies the effect of $X$ on $Y$ through $M$. Evidence that $ab$ is different from zero is consistent with mediation. Evidence that path $c$ is different from zero is not a requirement of 21st century mediation analysis. Correlation between $X$ and $Y$ is neither sufficient nor necessary to claim that $X$ affects $Y$. 
Moderation. The effect of $X$ on $Y$ can be said to be *moderated* if its size or direction is dependent on $M$. It tells us about the conditions that facilitate, enhance, or inhibit the effect, or for whom or what the effect is large vs. small, present versus absent, and so forth.

$M$ is depicted here to *moderate* the size of the effect of $X$ on $Y$, meaning that the size of the effect of $X$ on $Y$ depends on $M$. We say $M$ is the *moderator* of the $X \rightarrow Y$ relationship, or $X$ and $M$ *interact* in their influence on $Y$. 
“Simple linear moderation” is typically estimated by allowing $X$’s effect on $Y$ to be a linear function of $M$ (other forms of moderation are possible):

$$\hat{Y} = i_1 + (b_1 + b_3 M) X + b_2 M = i_1 + b_1 X + b_2 M + b_3 XM$$

In this model, the conditional effect of $X$ on $Y$, $\theta_{X \rightarrow Y}$, is $b_1 + b_3 M$:

$$\hat{Y} = i_1 + \theta_{X \rightarrow Y} X + b_2 M \quad \text{where} \quad \theta_{X \rightarrow Y} = b_1 + b_3 M$$

There is no effect of $X$ on $Y$ that one can reduce to a single estimate, for the effect of $X$ on $Y$ depends on $M$ unless $b_3$ is zero. An inference about the coefficient for $XM$ in the model is a widely used test of linear moderation.
Combining moderation and mediation analysis, at least in principle, is not new at all. Many have talked about it in the distant past (e.g., Judd & Kenny, 1981; James & Brett, 1984; Baron and Kenny, 1986). It goes by various names that often confuse, including “moderated mediation” and “mediated moderation.”

**More recently:**

**Muller, Judd, and Yzerbyt (2005):** Describe analytical models and steps for assessing when “mediation is moderated” and “moderation is mediated.”

**Edwards and Lambert (2007):** Take a path analysis perspective and show how various effects in a simple mediation model can be conditioned on a third variable.

**Preacher, Rucker, and Hayes (2007):** Provide a formal definition of the *conditional indirect effect* and give formulas, standard errors, and a bootstrap approach for estimating and testing hypotheses about moderated mediation in five different models.

**MacKinnon and colleagues (e.g., Fairchild & MacKinnon, 2009):** Explicate various analytical approaches to testing hypotheses about mediated moderation and moderated mediation.

**Hayes (2013) and Hayes and Preacher (2013):** Introduce the term “conditional process modeling” and (in Hayes and Preacher, 2013) take a structural equation modeling approach to estimating the contingent nature of direct and indirect effects.
Examples in substance use research

As a result of these recent discussions and the analytical approaches described therein, models that combine moderation and mediation are seen in the literature with increasing frequency, including in alcoholism research.


The indirect effect of $X$ on $Y$ through $M$ is estimated as the product of the $a$ and $b$ paths. But what if size of $a$ or $b$ (or both) depends on another variable (i.e., is moderated)? If so, then the magnitude of the indirect effect therefore depends on a third variable, meaning that “mediation is moderated”.

When $a$ or $b$ is moderated, it is sensible then to estimate “conditional indirect effects”—values of indirect effect conditioned on values of the moderator variable that moderates $a$ and/or $b$.

Direct effects can also be conditional. For instance, in the above, $W$ moderates $X$’s direct effect on $Y$. 

**Example inspired by ...**

**MBRP**: Randomly assigned to treatment as usual (0) or mindfulness-based relapse prevention therapy (1)

**BDIP**: Beck Depression Inventory scores immediately following completion of therapy.

**CRAVE2**: Score on the Penn Alcohol Craving Scale at 2 month follow-up.

**USE4**: Alcohol and other substance use at 4-month follow-up measured with the Timeline Follow-Back.

Covariates in the model included depression at start of therapy (BDIO), craving at baseline (CRAVO0) and hours in treatment (TREATHRS).
The model

Question: Do the skills acquired through MBRP therapy moderate craving as the mechanism through which negative affect influences alcohol and other substance use? This is a “first stage” moderated mediation model that also allows for the direct effect of $X$ to be moderated.
A conditional process model with a common moderator of the first stage path of the $X \rightarrow M \rightarrow Y$ indirect effect (the mechanism) as well as the direct effect of $X$ on $Y$.

This model is estimated (using OLS, for example) as:

$\hat{M} = i_1 + a_1 X + a_2 W + a_3 XW + ...$

$Y = i_2 + c_1'X + c_2'W + c_3'XW + bM + ...$

or equivalently

$\hat{M} = i_1 + (a_1 + a_3 W) X + a_2 W + ...$

$\hat{Y} = i_2 + (c_1' + c_3'W) X + c_2'W + bM + ...$

Using my “theta notation”:

$\hat{M} = i_1 + \theta_{X \rightarrow M} X + a_2 W + ...$

$\hat{Y} = i_2 + \theta_{X \rightarrow Y} X + c_2'W + bM + ...

where $\theta_{X \rightarrow M} = a_1 + a_3 W$ and $\theta_{X \rightarrow Y} = c_1' + c_3'W$
The conditional indirect effect

This model is estimated as:

\[
\hat{M} = i_1 + a_1 X + a_2 W + a_3 XW + ...
\]

\[
Y = i_2 + c_1' X + c_2' W + c_3' XW + bM + ...
\]

or equivalently

\[
\hat{M} = i_1 + \theta_{X\to M} X + a_2 W + ...
\]

\[
\hat{Y} = i_2 + \theta_{X\to Y} X + c_2' W + bM + ...
\]

\[
\theta_{X\to M} = a_1 + a_3 W \quad \text{and} \quad \theta_{X\to Y} = c_1' + c_3' W
\]

The indirect effect of \( X \) on \( Y \) through \( M \) is the product of the effect of \( X \) on \( M \) and the effect of \( M \) on \( Y \): \( \omega_M = \theta_{X\to M} b = (a_1 + a_3 W)b = a_1 b + a_3 bW \). This is a function of \( W \). Plug in a value of \( W \) and you get the “conditional indirect effect” of \( X \) on \( Y \) through \( M \), conditioned on that value of \( W \). An inference about that conditional indirect effect is an inference about “conditional” mediation. In this example, \( W \) is dichotomous, but it doesn’t have to be.
The conditional direct effect

\[ \hat{M} = i_1 + a_1 X + a_2 W + a_3 XW + \ldots \]
\[ Y = i_2 + c'_1 X + c'_2 W + c'_3 XW + bM + \ldots \]

This model is estimated as:

\[ \hat{M} = i_1 + \theta_{X\rightarrow M} X + a_2 W + \ldots \]
\[ \hat{Y} = i_2 + \theta_{X\rightarrow Y} X + c'_2 W + bM + \ldots \]
\[ \theta_{X\rightarrow M} = a_1 + a_3 W \quad \text{and} \quad \theta_{X\rightarrow Y} = c'_1 + c'_3 W \]

The direct effect of \( X \) on \( Y \) through \( M \) is \( \theta_{X\rightarrow M} = c'_1 + c'_3 W \). This is a function of \( W \). Plug in a value of \( W \) and you get the “conditional direct effect” of \( X \) on \( Y \). An inference about that conditional direct effect is an inference about whether \( X \) affects \( Y \) independent of the mechanism through \( M \), conditioned on that value of \( W \).
Easy to do with software you are (probably) already using

The PROCESS macro for SPSS and SAS is turn-key and easy to use but less flexible because the user is constrained to models PROCESS is programmed to estimate. PROCESS is freely available at www.afhayes.com

**SPSS:**
```
process vars=crave2 use4 bdip mbrp crave0 bdi0 treathrs/y=use4/m=crave2 /x=bdip/w=mbrp/model=8/boot=10000.
```

**SAS:**
```
%process (data=meditate,vars=crave2 use4 bdip mbrp crave0 bdi0 treathrs, y=use4,m=crave2,x=bdip,w=mbrp, model=8,boot=10000);
```

Mplus can also be used. It requires more programming skill but is more versatile with more benefits and fewer limitations.
SPSS

```bash
process var=crave2 use4 bdip mbpr crave0 bdi0 treathrs y=use4/m=crave2/x=bdip
```

SAS

```bash
proc (data=mediate, var=crave2 use4 bdip mbpr crave0 bdi0 treathrs, y=use4, m=crave2, x=bdip, =mbpr, model=0, boot=10000);
```

********** PROCESS Procedure for SPSS Release 2.12 **********

Written by Andrew F. Hayes, Ph.D.  www.afhayes.com


```
Model  = 8
Y  = use4
X  = bdip
M  = crave2
W  = mbpr

Statistical Controls:
CTRLCL= crave0 bdi0 treathrs

Sample size 468

Conclusion: crave2
```

```
Outcome: use4

Model Summary

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Interactions:

int_1 bdip X mbpr
```

Output: use4

Model Summary

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Model

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Interactions:

int_2 bdip X mbpr

************************* DIRECT AND INDIRECT EFFECTS *************************

Conditional direct effect(s) of X on Y at values of the moderator(s):

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Conditional indirect effect(s) of X on Y at values of the moderator(s):

Mediator

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Values for quantitative moderators are the mean and plus/minus one SD from mean.

Values for dichotomous moderators are the two values of the moderator.

************************* INDEX OF MODERATED MEDIATION *************************

Mediator

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When the moderator is dichotomous, this is a test of equality of the conditional indirect effects in the two groups.

************************** ANALYSIS NOTES AND WARNINGS **************************

Number of bootstrap samples for bias corrected bootstrap confidence intervals: 10000

Level of confidence for all confidence intervals in output: 95.00
**PROCESS output**

*************** DIRECT AND INDIRECT EFFECTS ***********************

\[ W (c_1' + c_3'w) \]

Bootstrap confidence intervals

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<th>Effect</th>
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Conditional indirect effect(s) of \( X \) on \( Y \) at values of the moderator(s):

\[ W (a_1 + a_3w)b \]

Bootstrapping confidence intervals

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Values for quantitative moderators are the mean and plus/minus one SD from mean. Values for dichotomous moderators are the two values of the moderator.

*************** INDEX OF MODERATED MEDIATION ***********************

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Difference between conditional indirect effects (with bootstrap confidence interval)

When the moderator is dichotomous, this is a test of equality of the conditional indirect effects in the two groups.

Direct and indirect effects of depression on substance use are positive and statistically different from zero among those given therapy as usual. No direct or indirect effects of depression on substance use among those given MBRP therapy. The indirect effect through craving differs between the two groups---”moderated mediation”
Some other examples

We just examined a “first stage” model. But moderation can occur in the “second stage” of the mechanism as well:


....or a variable can moderate both stages of the mechanism.

There are many possibilities. The math is different, but the principles are the same.

An intriguing possibility

A causal agent modifying the operation of its own mechanism by which it affects an outcome.

This model is estimated as:

\[ \hat{M}_1 = i_1 + aX \]
\[ \hat{Y} = i_2 + c'X + b_1M + b_2XM \]

or equivalently

\[ \hat{M}_1 = i_1 + aX \]
\[ \hat{Y} = i_2 + c'X + (b_1 + b_2X)M \]

The effect of \( X \) on \( M \) is just “\( a \)”, but the effect of \( M \) on \( Y \) depends on \( X \): \( b_1 + b_2X \).

The indirect effect of \( X \) is the product of these effects: \( a(b_1 + b_2X) = ab_1 + ab_2X \) and so depends on \( X \). This makes sense to do only if \( X \) is not dichotomous.
Multiple mechanisms modeled simultaneously


Why estimate such a model?

- Many causal effects probably operate through multiple mechanisms simultaneously. Better to estimate a model consistent with such real-world complexities.

- If your proposed mediator is correlated with the “real” mediator but not caused by the independent variable, a model with only your proposed mediator in it will be a misspecification and will potentially misattribute the process to your proposed mediator rather than the real mediator—“epiphenomenality.”

- Different theories may postulate different mediators as mechanisms. Including them all in a model simultaneously allows for a formal statistical comparison of indirect effects representing different theoretical mechanisms.

- When combined with moderation, allows for the modeling of different mechanisms for different people defined by different values of a moderator.
An interesting extension

Mechanisms might be different for different types of people. For some types, mechanism 1 may be dominant, whereas for other types, mechanism 2 may dominate. For example:

A conditional process model with a common moderator of both of the first stage paths of the mechanism.

This model is estimated as:

\[
\hat{M}_1 = i_1 + a_{11}X + a_{12}W + a_{13}XW \\
\hat{M}_2 = i_2 + a_{21}X + a_{22}W + a_{23}XW \\
\hat{Y} = i_3 + c'X + b_1M_1 + b_2M_2
\]

or equivalently

\[
\hat{M}_1 = i_1 + (a_{11} + a_{13}W)X + a_{12}W \\
\hat{M}_2 = i_2 + (a_{21} + a_{23}W)X + a_{22}W \\
\hat{Y} = i_3 + c'X + b_1M_1 + b_2M_2
\]
An interesting extension

Mechanisms might be different for different types of people. For some types, mechanism 1 may be dominant, whereas for other types, mechanism 2 may dominate.

A conditional process model with a common moderator of both of the first stage paths of the mechanism.

\[ \hat{M}_1 = i_1 + (a_{11} + a_{13}W)X + a_{12}W \]
\[ \hat{M}_2 = i_2 + (a_{21} + a_{23}W)X + a_{22}W \]
\[ \hat{Y} = i_3 + c'X + b_1M_1 + b_2M_2 \]

Indirect effect of \( X \) on \( Y \) through \( M_1 \) depends on \( W \):
\[ \omega_1 = (a_{11} + a_{13}W)b_1 \]
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An interesting extension

Mechanisms might be different for different types of people. For some types, mechanism 1 may be dominant, whereas for other types, mechanism 2 may dominate.

A conditional process model with a common moderator of both of the first stage paths of the mechanism.

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\[
\begin{align*}
\hat{M}_1 &= i_1 + (0.40 - 0.40W)X + a_{12}W \\
\hat{M}_2 &= i_2 + (0.00 + 0.60W)X + a_{22}W \\
\hat{Y} &= i_3 + c'X + 0.50M_1 + 0.30M_2
\end{align*}
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Indirect effect of \(X\) on \(Y\) through \(M_1\) depends on \(W\):

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\[
\omega_1 = 0.20 - 0.20W
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Indirect effect of \(X\) on \(Y\) through \(M_2\) depends on \(W\):

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\omega_2 = (a_{21} + a_{23}W)b_2
\]

\[
\omega_2 = (0.00 + 0.60W)0.30
\]

\[
\omega_2 = 0.00 + 0.18W
\]
An interesting extension

Mechanisms might be different for different types of people. For some types, mechanism 1 may be dominant, whereas for other types, mechanism 2 may dominate.

For people of type A (e.g., $W = 0$) $X$ affects $Y$ through $M_1$ but not through $M_2$.

**Indirect effect of $X$ on $Y$ through $M_1$ when $W = 0$:**

$$\omega_1 = (a_{11} + a_{13}W)b_1$$

$$\omega_1 = (0.40 - 0.40 \times 0) 0.50$$

$$\omega_1 = 0.20 - 0.20 \times 0 = 0.20$$

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\[
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Mechanisms might be different for different types of people. For some types, mechanism 1 may be dominant, whereas for other types, mechanism 2 may dominate.

For people of type B (e.g., $W = 1$) $X$ affects $Y$ through $M_2$ but not through $M_1$.

Indirect effect of $X$ on $Y$ through $M_1$ when $W = 1$:
\[ \omega_1 = (a_{11} + a_{13}W)b_1 \]
\[ \omega_1 = (0.40 - 0.40 \times 1)0.50 \]
\[ \omega_1 = 0.20 - 0.20 \times 1 = 0.00 \]

Indirect effect of $X$ on $Y$ through $M_2$ when $W = 1$:
\[ \omega_2 = (a_{21} + a_{23}W)b_2 \]
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$$\omega_2 = (0.00 + 0.60 \times 1)0.30$$
$$\omega_2 = 0.00 + 0.18 \times 1 = 0.18$$
In closing...

These are slides at www.afhayes.com/public/mobc.pdf

- All causal effects operate through some kind of mechanism—a causal chain of events. But all effects are contingent on something.

- Mechanisms that are contingent can be modeled if we understand or can at least hypothesize something about those contingencies.

- Quantifications of mechanisms (indirect effects) can be modeled as functions of other variables (moderators).

- Simple combinations of moderation and mediation can be put together to yield complex models that are yet fairly simple to estimate and interpret.

- Statistical tools exist to make the modeling easy, and people are beginning to do this in earnest in many areas of research, including substance use.

- Learning resources are scattered throughout the methodology journals. The advice they offer is often inconsistent, sometimes dated.
Some places to go for help

www.afhayes.com
Some places to go for help

www.statisticalhorizons.com

Mediation and Moderation
A 5-Day Seminar Taught by Andrew Hayes, Ph.D.

Read reviews of this course

This seminar focuses on two topics in causal analysis that are closely related and often confused. Suppose we have three variables, X, M, and Y. We say that M is a mediator of the effect of X on Y if X carries its influence on Y at least partly by influencing M, which then influences Y. This is also known as an indirect effect of X on Y through M. On the other hand, we say that M moderates the effect of X on Y if that effect varies in size, sign, or strength as a function of M. This is also known as interaction.

Although these concepts are fairly simple, the statistical issues that arise in estimating and testing mediation and moderation effects turn out to be rather complex and subtle. Andrew Hayes has been among the leading recent contributors to the literature on these methods. He has developed powerful new methods for estimating mediation and moderation effects and special software tools that can be used with SAS or SPSS.

In this seminar, you will learn about the underlying principles and the practical applications of these methods. The seminar is divided roughly into three parts:

1. Partitioning effects into direct and indirect components, and how to quantify and test hypotheses about indirect effects.
2. Estimating, testing, probing, and visualizing interactions in linear models.
3. Integrating moderation and mediation by discussing how to estimate conditional indirect effects, determine whether an indirect effect is moderated (moderated mediation) and whether moderated effects are mediated (mediated moderation).

Computer applications will focus on the use of OLS regression and computational modeling tools for SPSS and SAS (including the PROCESS add on developed by Hayes). When appropriate, some Mplus code will be provided for those interested, but structural equation modeling and Mplus will

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SEMINAR INFORMATION
Monday July 14, 2014 9:00 AM - Friday July 18, 2014 5:00 PM (Eastern Time)
The Helb Commerce Square
2001 Market Street -- Hadron Room
Philadelphia, Pennsylvania 19103
United States
View Map

CONTACT INFORMATION
Phone: 610-642-1841
Fax: 415-919-1220
Email: info@statisticalhorizons.com

PAYMENT INSTRUCTIONS
The fee of $1695 includes all course materials. The early registration fee of $1495 is available until June 16.
PayPal and all major credit cards are accepted.
Our Tax ID number is 26-4576270.

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